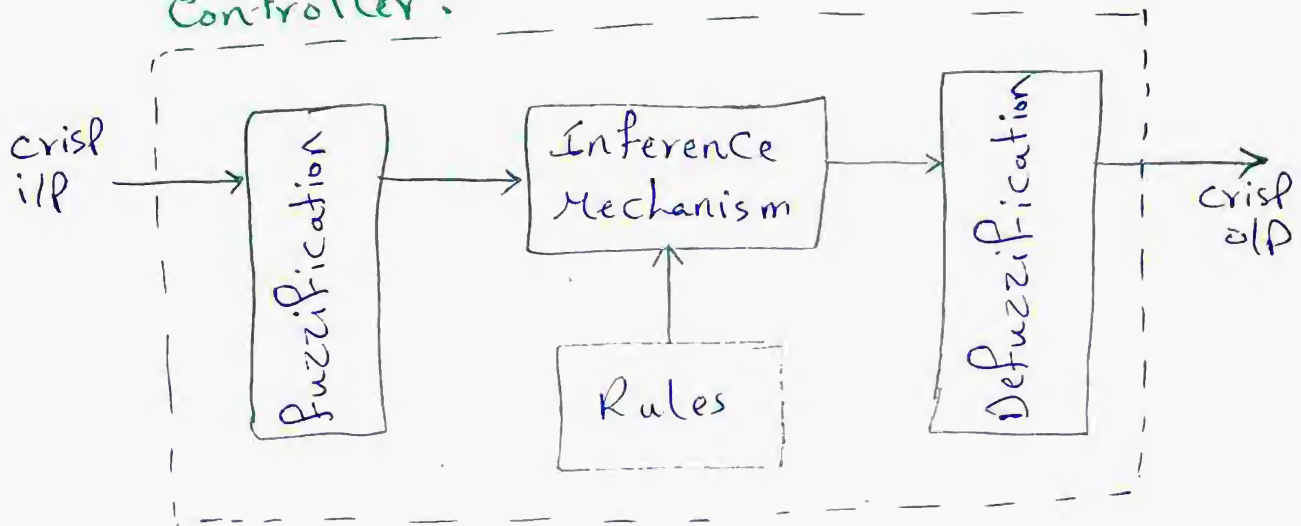


## Fuzzy: Final 2016

Q1

a) Explain the main structure of Fuzzy Controller.



### 1) Fuzzification

↳ Convert crisp values of fuzzy controller input into fuzzy input sets.

### 2) Rules & Inference Mechanism

Rules: set of IF-then statements, that governs the performance of controller.

Inference mechanism: emulates the expert's decision making in interpreting & apply knowledge about best to control plant.

### 3) Defuzzification:-

↳ Inverse process of fuzzification (convert fuzzy quantity into crisp value)

11

Q. b) what are the main differences between Mamdani and TSK Fuzzy Controller:

1) The difference in the rules

a. In Mamdani: rules obtained from an experienced human operator.

For ex

R1: if  $x_1$  is  $A_1$  and  $x_2$  is  $B_1$ , then  $y$  is  $C_1$

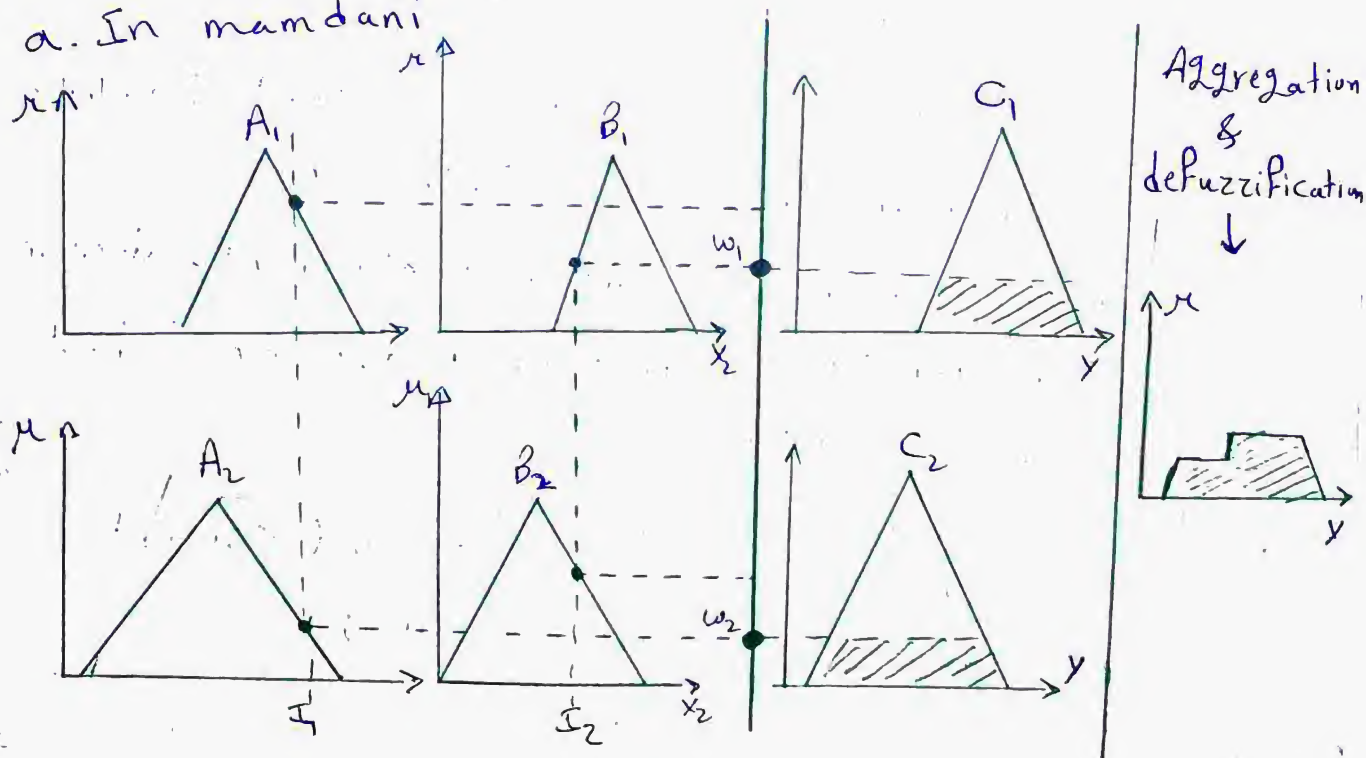
b. In TSK (rules) ~~are~~ linear function which is combination of input variables + constant term

[Ex]

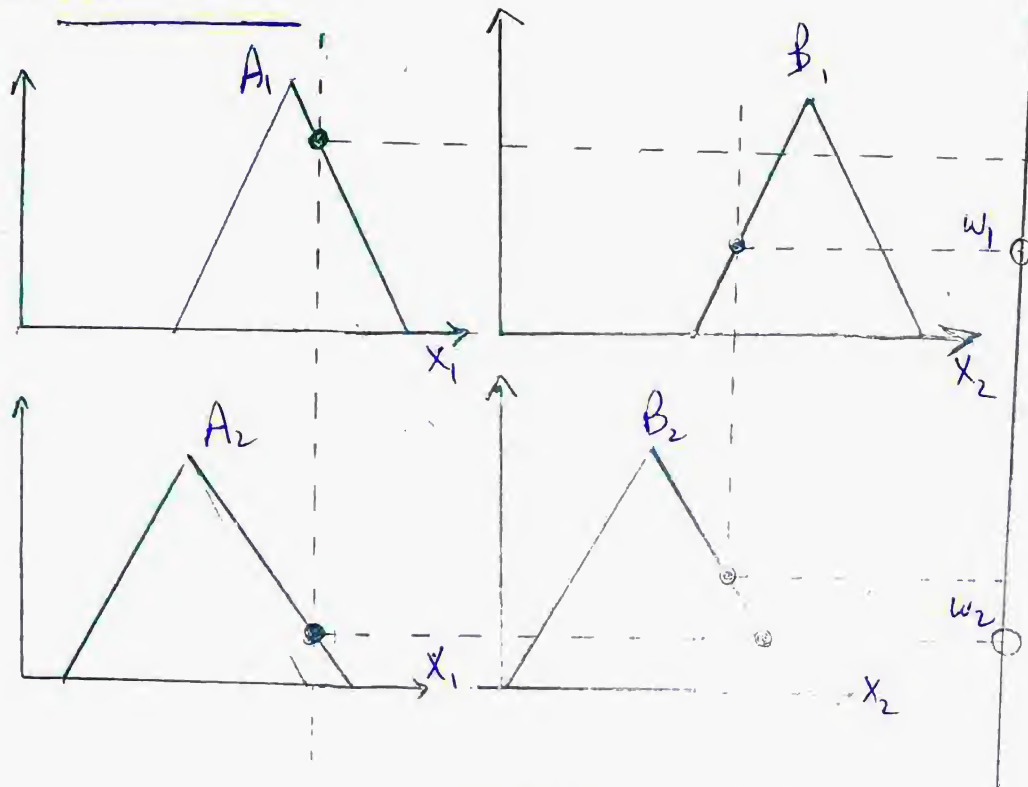
R1: if  $x_1$  is  $A_1$  &  $x_2$  is  $B_1$ , then  $y = p_1 x_1 + q_1 x_2 + r_1$

2) The difference in way of apply defuzzification

a. In Mamdani



b. In TsK



$$Y_1 = P_1 X_1 + Q_1 X_2 + R_1$$

$$Y_2 = P_2 X_1 + Q_2 X_2 + R_2$$

weighted average

$$Y = \frac{w_1 Y_1 + w_2 Y_2}{w_1 + w_2}$$

defuzzification ← معك لو عطلت معاك في جزء ال  
تقول انه الغزو هو كده :

1) way of defuzzification

a. In Mamdani

\* center of gravity. \* weighted average

\* Mean-max membership.

b. In TsK

↳ weighted average.



Q. c) Explain the Following.

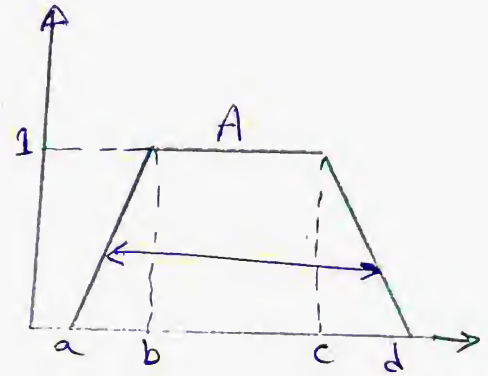
support - core - singleton

1. support

↳ elements of fuzzy set where its MF degree  $\neq 0$

ex

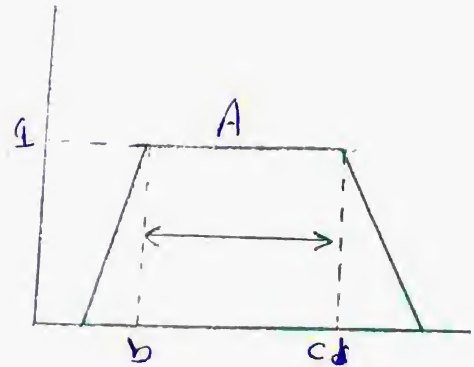
$$\text{support}(A) = [a, d]$$



2. Core

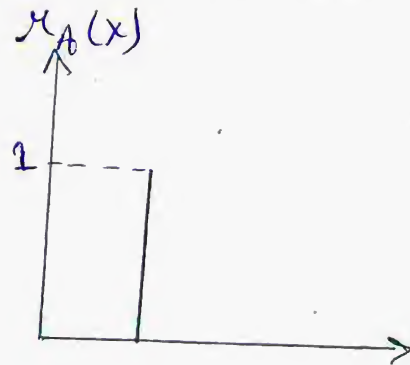
↳ elements of fuzzy set where its MF degree is equal to 1

$$\text{core}(A) = [b, c]$$



3. singleton

↳ when no. of elements of fuzzy set is equal to 1 with  $\mu = 1$ , it is called singleton.



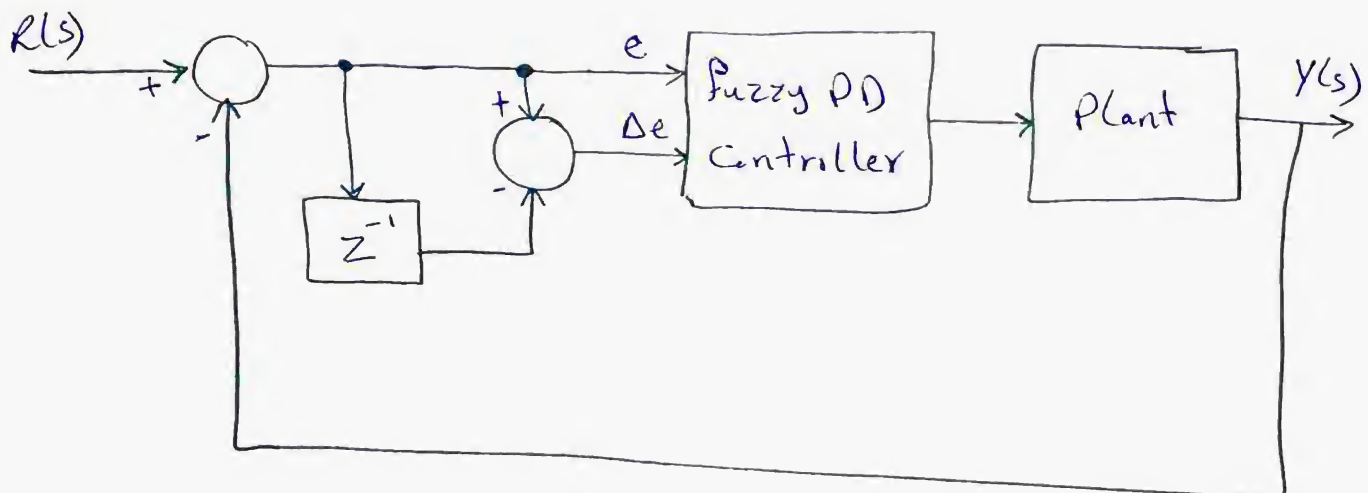
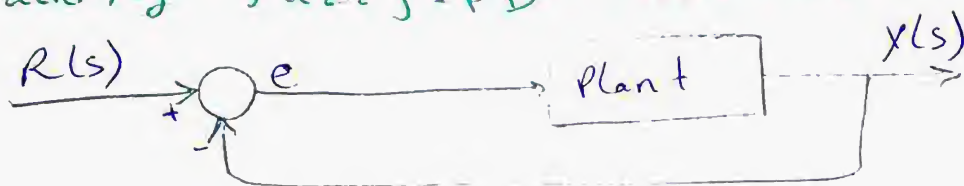
Q<sub>1</sub>. d) What are the advantages of using Fuzzy Controllers?

- 1) cheap in cost
- 2) Customizable
- 3) easy to design and implement.
- 4) more robustness.

لے مشمتاں کہ مطلوب نشر ہم ولا لا بس الشرح موجود فی

Lec. slide .01 Page 15, 16

Q<sub>1</sub>. e) redraw the following system after adding Fuzzy-PD Controller



**Q2** using the error signal ( $e$ ) and change of error ( $\Delta e$ )

design Fuzzy-PD controller with following specs:

a) no. of MFs for the inputs ( $e$  and  $\Delta e$ ) is 5.

b) " " " " " output ( $u$ ) is 3

c) use (NM, NS, Z, PS, PM) as labels of MFs for inputs ( $e$  &  $\Delta e$ )

d) " (NL, NM, NS, Z, PS, PM, PL) as labels of MFs for output ( $u$ )

e) the universe of discourse:

\*  $e \rightarrow$  From -4 to 4

\*  $\Delta e \rightarrow$  From -1 to 1

\*  $u \rightarrow$  From -9 to 9

1) Draw MFs for input and output

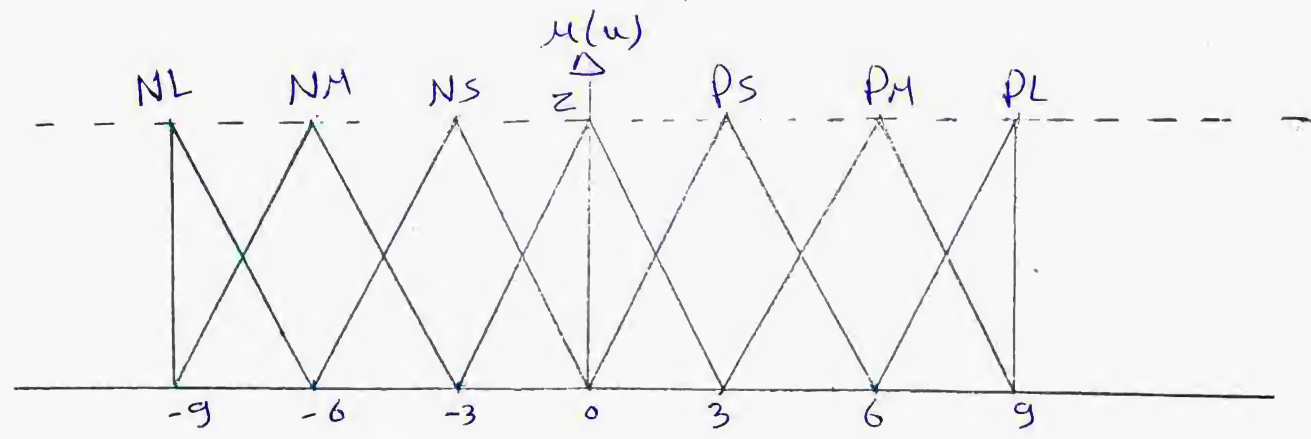
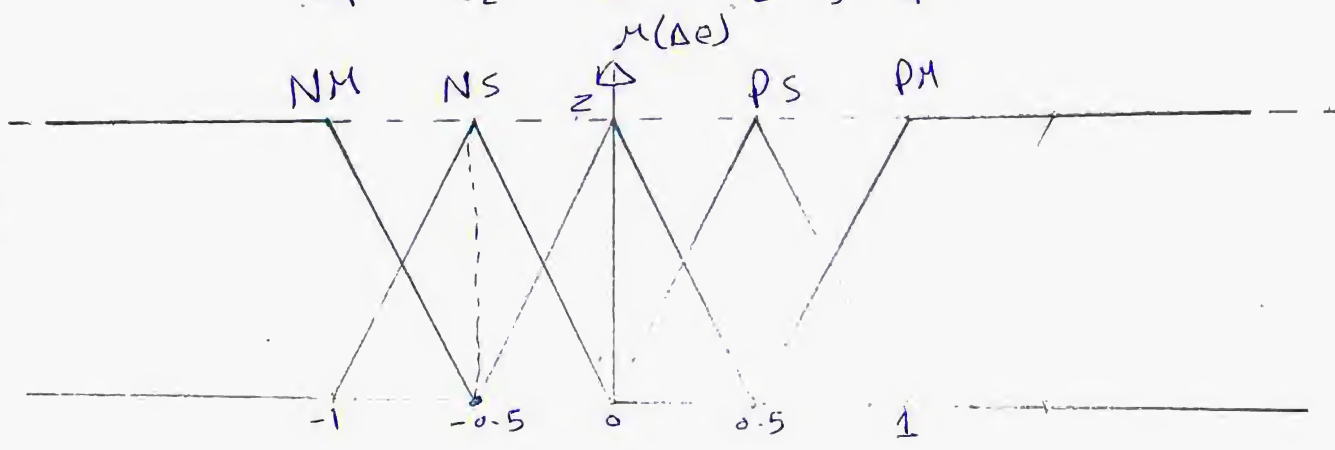
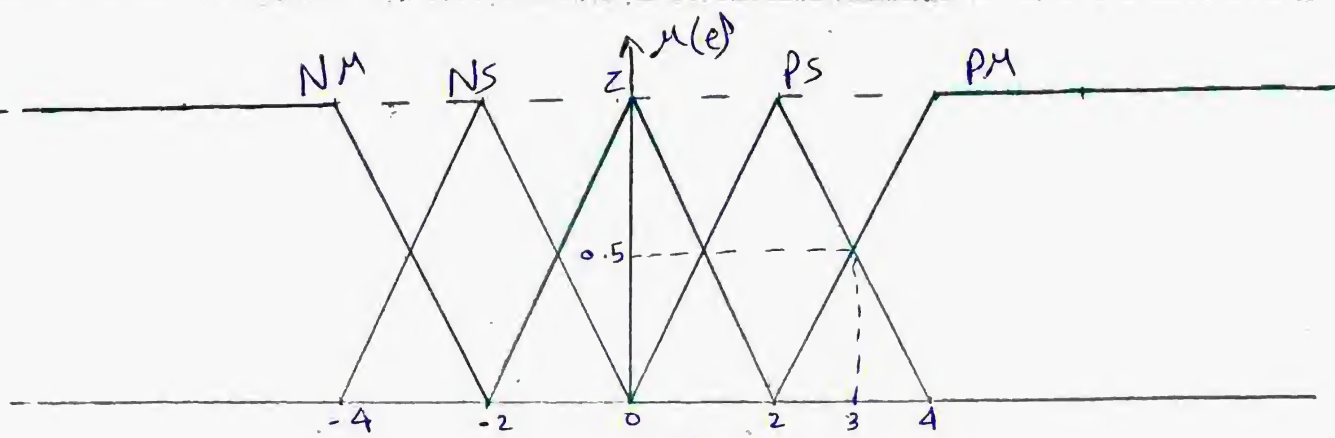
2) write the suitable rules.

3) Find the controller crisp output ( $u^{crisp}$ ) at

$e = 3$  and  $\Delta e = -0.5$

solution starts from

next page.



$\Delta e \backslash e$	NM	NS	Z	PS	PM
NM	PL	PL	PM	PS	Z
NS	PL	PM	PS	Z	NS
Z	PM	PS	Z	NS	NM
PS	PS	Z	NS	NM	NL
PM	Z	NS	NM	NL	NL

table of rules



$$e=3 \quad \& \quad \Delta e = -0.5$$

### 1) Fuzzification

$$e=3 \begin{cases} \rightarrow PS \text{ with } \mu_{PS}(e) = 0.5 \\ \rightarrow PM \text{ with } \mu_{PM}(e) = 0.5 \end{cases}$$

$$\Delta e = -0.5$$

$$\rightarrow NS \text{ with } \mu_{NS}(\Delta e) = +1$$

### 2) Fired rules

R1: if  $e$  is  $PS$  &  $\Delta e$  is  $NS$  then  $u$  is  $Z$

R2: if  $e$  is  $PM$  &  $\Delta e$  is  $NS$  then  $u$  is  $NS$

### 3) strength of Fired rules

$$\mu_{P_1} = \min \{ \mu_{PS}(e=3), \mu_{NS}(\Delta e = -0.5) \} = \min \{ 0.5, 1 \} = 0.5$$

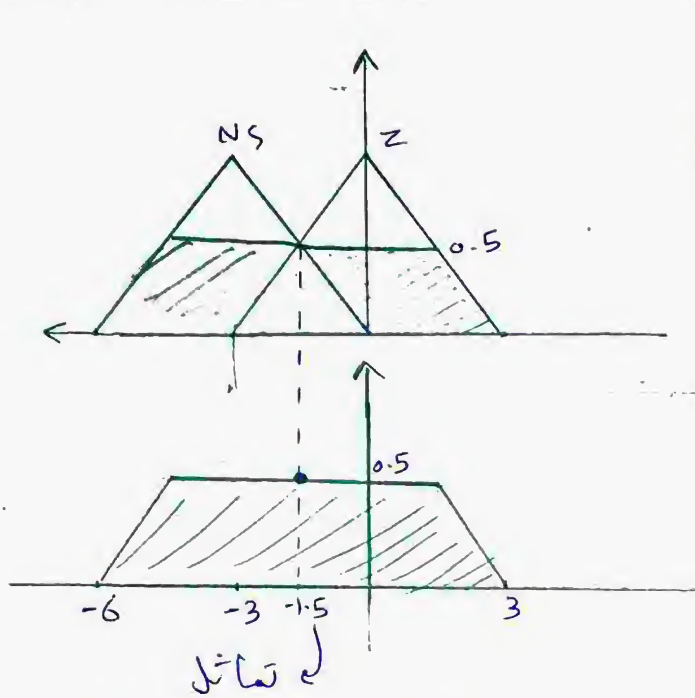
$$\mu_{P_2} = \min \{ \mu_{PM}(e=3), \mu_{NS}(\Delta e = -0.5) \} = \min \{ 0.5, 1 \} = 0.5$$

### 4) Forms of o/p Fuzzy sets:-

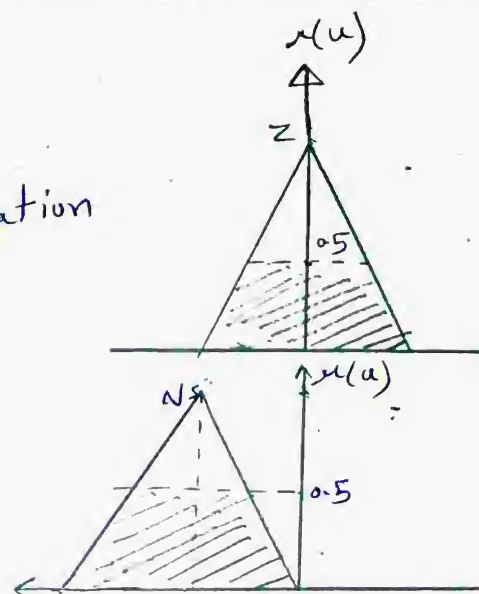
$$\mu_{Z_0}(u) = \min \{ \mu_{P_1}, \mu_Z(u) \} = \min \{ 0.5, \mu_Z(u) \}$$

$$\mu_{NS}(u) = \min \{ \mu_{P_2}, \mu_{NS}(u) \} = \min \{ 0.5, \mu_{NS}(u) \}$$





⑤ Aggregation



~~Aggregation~~

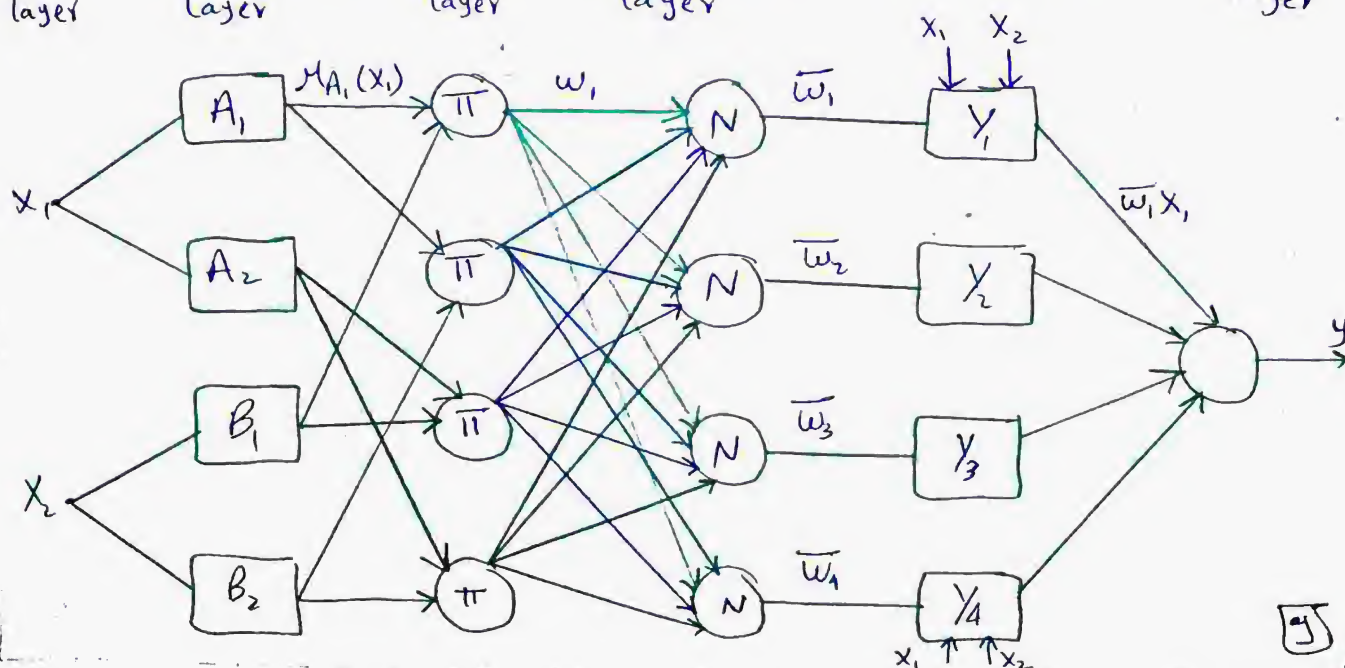
6) defuzzification

crisp  $u = -1.5$

Q3: a

The ANFIS structure is a multi-layered neural network, Explain these layers.

input layer    fuzzification layer    Rule layer    Normalization layer    defuzzification layer    sum layer



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## ANFIS Layers

### 1) Layer 0: input layer

↳ as inputs are applied to system.

### 2) Fuzzification layer: Layer 1

↳ apply inputs MFs & produce a degree of membership ( $\mu$ )

### 3) Layer 2: Rule layer

↳ Its output represents fire strength of rules.

↳ executes fuzzy of antecedent (if part)

### 4) Layer 3: normalization layer:

↳ o/p is ratio of firing strength of  $i$ th rules to sum of all firing strength rule.

$$\bar{w}_i = \frac{w_i}{w_1 + w_2 + w_3 + w_4}, \quad i=1, 2, 3, 4$$

### 5) Layer 4: defuzzification layer (then part)

↳ executes the consequent part of fuzzy rules.

↳ its o/p is product of normalized fired strength rule & its corresponding linear function in consequent part.

### 6) Layer 5: sum layer

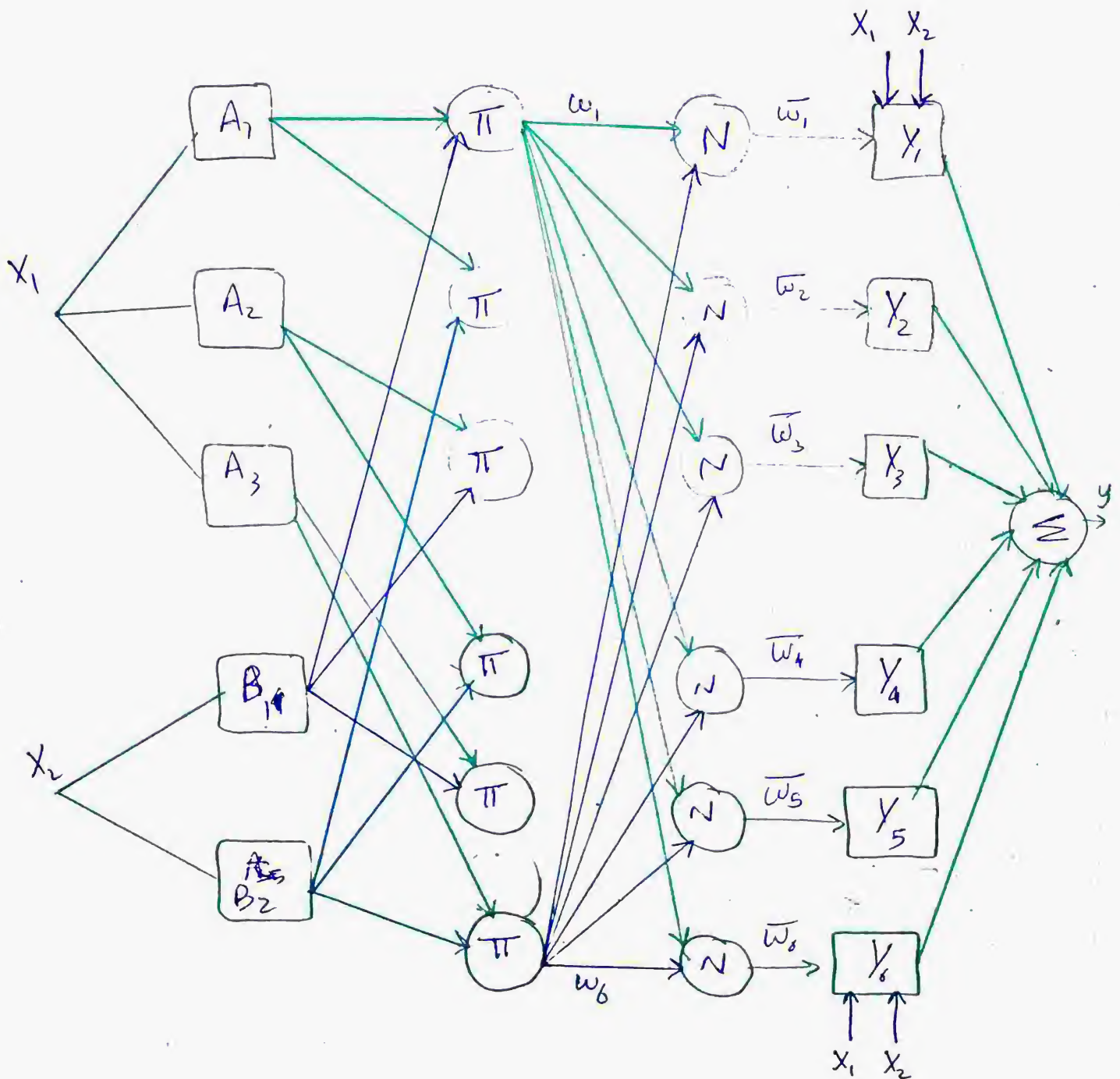
↳ computes total crisp output of fuzzy system

$$y^{\text{crisp}} = \frac{w_1 X_1 + w_2 X_2 + w_3 X_3 + w_4 X_4}{w_1 + w_2 + w_3 + w_4} = \frac{\bar{w}_1 X_1 + \bar{w}_2 X_2 + \bar{w}_3 X_3 + \bar{w}_4 X_4}{1}$$



Q3: b)

Draw the ANFIS structure for two input  $(x_1, x_2)$  when three MFs ( $A_1, A_2, A_3$ ) are used for input  $x_1$  and two MFs ( $B_1, B_2$ ) are used for  $x_2$





Q3.c) what are the Parameters to be optimized in ANFIS and what are the methods that can be used to optimize these Parameters?

sol

1. Parameters to be optimized:

- Premise Part (if-Part)
- Consequent Part (then-Part)

2. Methods used to optimize these Parameters:

a. derivative-based :

- backpropagation (BP)
- least squares estimate (LSE)
- Hybrid learning (HL)

b. derivative-free :

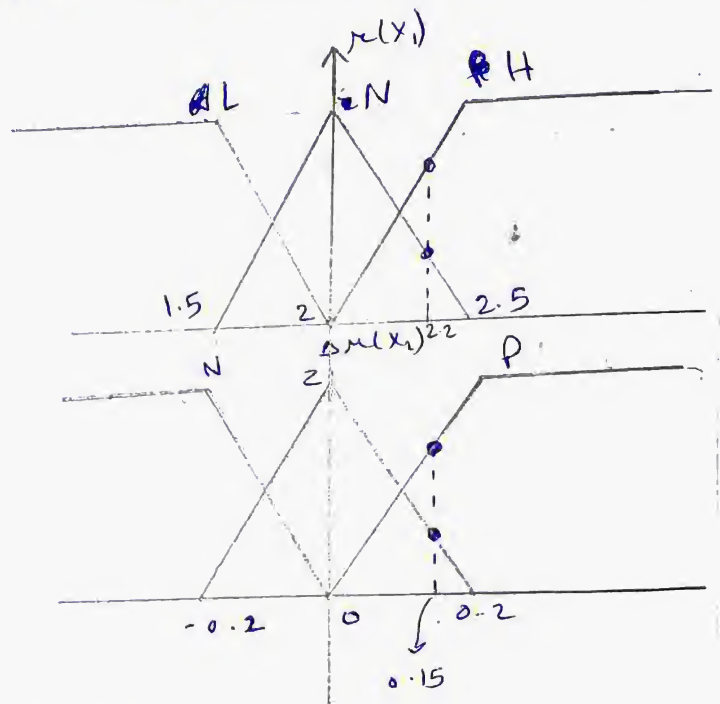
- genetic algorithm (GA)
- Particle swarm optimization (PSO)
- differential evolution (DE)
- Shuffled Frog leaping Algorithm (SFLA)
- artificial bee colony algorithm (ABC)

له لو نسييتهم معكم تكتب الرموز ما بين الأقواس

لكم يفضل تكتبهم طبعاً

Q<sub>3</sub>.d) A TSK Fuzzy controller is designed for a level control process, with two ~~two~~ inputs  $X_1$  (represent the level) &  $X_2$  (change in level). The output of the controller  $u$  (represent the valve position).  
The MFs for  $X_1, X_2$  rules of controller:

$X_1 \backslash X_2$	N	Z	P
L	$Y_1$	$Y_1$	$Y_1$
N	$Y_2$	$Y_2$	$Y_3$
H	$Y_4$	$Y_4$	$Y_4$



Where:  $Y_1 = \cancel{4X_1} - 0.25X_2 + 0.05$

$$Y_2 = X_1 - 0.1 X_2$$

$$Y_3 = 0.5X_1 - 0.1 X_2$$

$$Y_4 = 0.2 X_1 - X_2$$

Find the controller crisp output ( $u^{crisp}$ )

when  $X_1 = 2.2$  and  $X_2 = 0.15$

## 1) Fuzzification

•  $X_1 = 2.2$

$$\begin{cases} \rightarrow H & \text{with } \mu_H(X_1) = 0.4 \\ \rightarrow N & \text{with } \mu_N(X_1) = 0.6 \end{cases}$$

$X_2 = 0.15$

$$\begin{cases} \rightarrow P & \text{with } \mu_P(X_2) = 0.75 \\ \rightarrow Z & \text{with } \mu_Z(X_2) = 0.25 \end{cases}$$

## 2) Fired rules

$R_1$ : if  $X_1$  is  $H$  &  $X_2$  is  $P$  then  $Y_4 = 0.2X_1 - X_2$

$R_2$ : if  $X_1$  is  $H$  &  $X_2$  is  $Z$  then  $Y_4 = 0.2X_1 - X_2$

$R_3$ : if  $X_1$  is  $N$  &  $X_2$  is  $P$  then  $Y_3 = 0.5X_1 - 0.1X_2$

$R_4$ : if  $X_1$  is  $N$  &  $X_2$  is  $Z$  then  $Y_2 = X_1 - 0.1X_2$

For  $X_1 = 2.2$  &  $X_2 = 0.15$

$Y_4 = 0.29$

$Y_3 = 1.085$

$Y_2 = 2.185$

$X_1 \downarrow H$  معادلة الخط

$$\frac{\mu - 0}{X_1 - 2} = \frac{1 - 0}{2.5 - 2} = \frac{1}{0.5}$$

$$0.5\mu = X_1 - 2 \quad (X_1 = 2.2)$$

$$\mu = \frac{0.2}{0.5} = 0.4$$

$X_1 \downarrow N$  معادلة الخط

$$\frac{\mu - 1}{X_1 - 2} = \frac{0 - 1}{2.5 - 2}$$

$$0.5(\mu - 1) = 2 - X_1$$

$$\mu = 0.6$$

$X_2 \downarrow P$  معادلة الخط

$$\frac{\mu - 0}{X_2 - 0} = \frac{1 - 0}{0.2 - 0}$$

$$0.2\mu = X_2 = 0.15$$

$$\mu = 0.75$$

$X_2 \downarrow Z$  معادلة الخط

$$\frac{\mu - 1}{X_2 - 0} = \frac{0 - 1}{0.2 - 0}$$

$$0.2(\mu - 1) = -X_2$$

$$\mu = 0.25$$

المعادلات معاً  
التوضيح فقط



### \* The Fired rules

$R_1$ : if  $X_1$  is H &  $X_2$  is P then  $Y_4 = 0.29$

$R_2$ : if  $X_1$  is H &  $X_2$  is Z then  $Y_4 = 0.29$

$R_3$ : if  $X_1$  is N &  $X_2$  is P then  $Y_3 = 1.085$

$R_4$ : if  $X_1$  is N &  $X_2$  is Z then  $Y_2 = 2.185$

### 3) strength of Fired rules

$$R_1: \mu_{P_1} = \min \{ \mu_H(X_1), \mu_P(X_2) \} = \min \{ 0.4, 0.75 \} = 0.4$$

$$R_2: \mu_{P_2} = \min \{ \mu_H(X_1), \mu_Z(X_2) \} = \min \{ 0.4, 0.25 \} = 0.25$$

$$R_3: \mu_{P_3} = \min \{ \mu_N(X_1), \mu_P(X_2) \} = \min \{ 0.6, 0.75 \} = 0.6$$

$$R_4: \mu_{P_4} = \min \{ \mu_N(X_1), \mu_Z(X_2) \} = \min \{ 0.6, 0.25 \} = 0.25$$

### 4) Aggregation and defuzzification:

$$R_1: Y_4 = 0.29 \quad \text{with } w_1 = 0.4$$

$$R_2: Y_4 = 0.29 \quad \text{with } w_2 = 0.25$$

$$R_3: Y_3 = 1.085 \quad \text{with } w_3 = 0.6$$

$$R_4: Y_2 = 2.185 \quad \text{with } w_4 = 0.25$$

using weighted average method

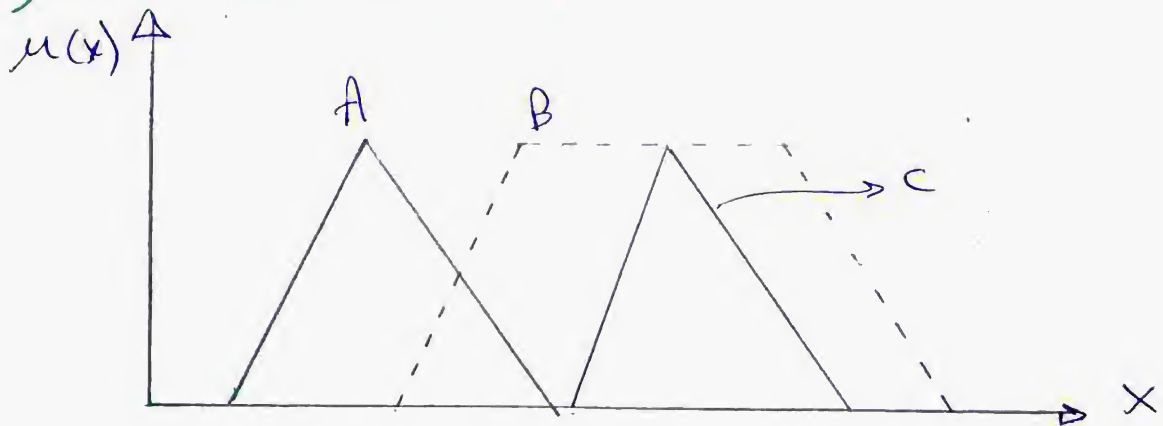
$$y^{\text{crisp}} = \frac{w_1 Y_1 + w_2 Y_2 + w_3 Y_3 + w_4 Y_4}{w_1 + w_2 + w_3 + w_4}$$

$$= \frac{w_1 Y_1 + w_2 Y_2 + w_3 Y_3 + w_4 Y_4}{w_1 + w_2 + w_3 + w_4}$$

$$= \frac{0.4 \times 0.29 + 0.25 \times 0.29 + 0.6 \times 1.085 + 0.25 \times 2.185}{0.4 + 0.25 + 0.6 + 0.25}$$

$$y^{\text{crisp}} = 0.92$$

Q4.a) For the Following



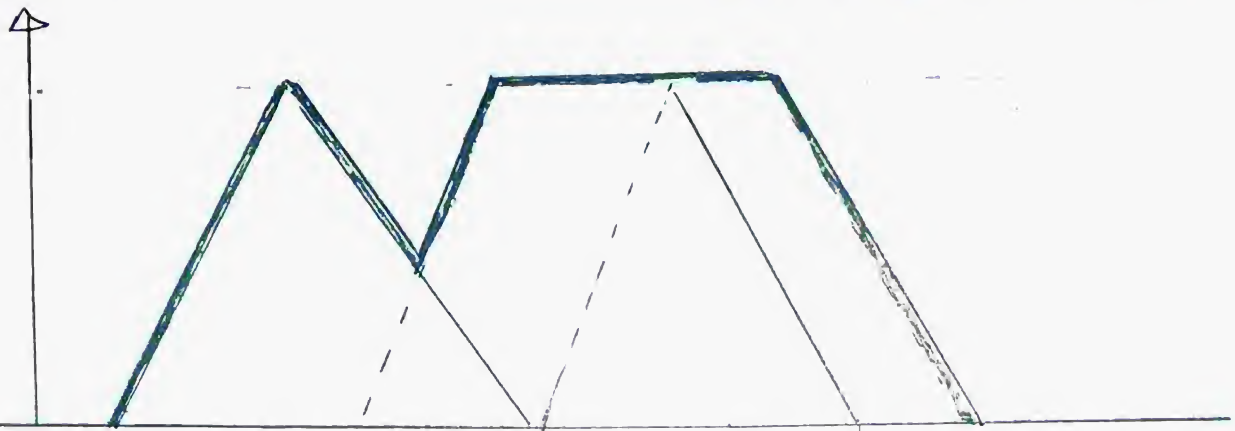
Find  $A \cup B \cup C$

Find  $A \cap B \cap C$

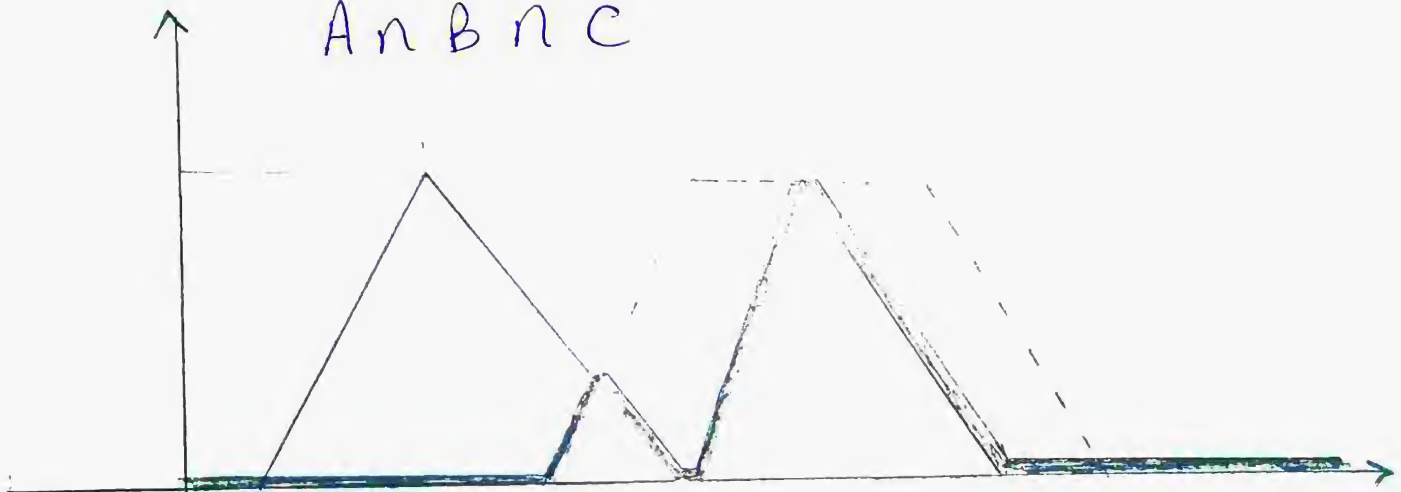
Find  $\bar{B}$

$A \cup B \cup C$

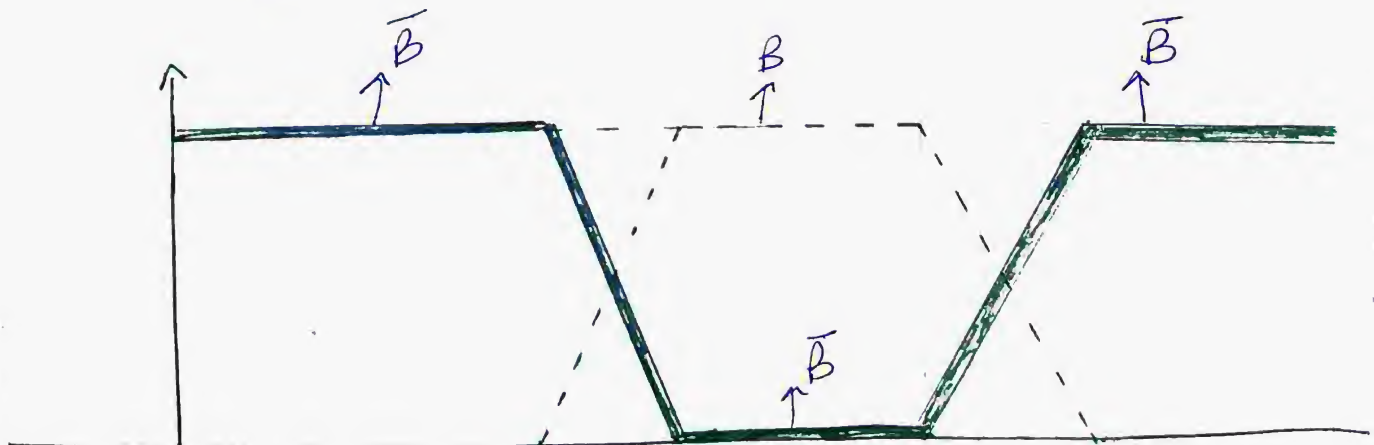
الخط التقيد هو الاتحاد



$A \cap B \cap C$



$\bar{B}$





Q4. b) write briefly about an application that used an application that uses Fuzzy Control Field & explain the inputs and outputs that used in this app.

له يفهل كل طالب يد ( Presentation )  
التي قدمها

هو تاجر ال ( o/p , i/p )

جدول ال ( rules ) لوائمه .

وشرح ال system